By reviewing the Scosine and the Ssine we see some interesting mathematic relationships:

$$\cos(\theta) = \frac{x}{hypotenuse}\ (if\ right\ triangle) = \frac{1}{L_{Scosine} + radius}\ (for\ any\ angle) = 3\ (in\ our\ example)$$

$$\sin(\theta) = \frac{y}{hypotenuse}\ (if\ right\ triangle) = \frac{1}{L_{Ssine} + radius}$$

$$In\ a\ right\ triangle;\ \sin(\theta_1) + \sin(\theta_2) = 1$$

$$L = hypotenuse \text{ - } x = \frac{|\cos(\theta_1) \text{ - } \cos(\theta_2)|}{\cos(\theta_2)} * radius = \frac{|1 \text{ - } \cos(\theta_2)|}{\cos(\theta_2)} * radius\ (when\ right\ angle)$$

There is also a inverse relationship between the sine of the Scosine value and the cosine of the Ssine value:

Needs tested refer to picture of Ssine and Scosine.

$$L_{Scosine} * \sin(\theta_2) = \frac{1}{L_{Scosine}} * \cos(\theta_2)$$

Now these equations are very useful. However we need to find the right combination of x and y in order to get a useful value. After several hundred equations, this is the best I have so far:

$$y = \frac{x}{\frac{1}{L_{Scosine} + radius}} + [L * \sin(\theta_2)]$$

test with value of 3:

$$y \ = 3x + [\frac{1}{L} * \cos(\theta_2)]$$

$$since\ \frac{1}{L} * \cos(\theta_2) = \sin(\theta_2)$$

$$y = 3x + [[\frac{1}{3\text{-}x}] * \frac{x}{3}]$$

$$since\ L = 3 \text{ - } x$$

$$y = 3x + \frac{x}{9 \text{ - } 3}$$

add fractions

$$y = \frac{3x\ (9 \text{ - } 3x^2)}{9 \text{ - } 3x} + \frac{x}{9 \text{ - } 3x}$$

$$y = \frac{\text{-}3x^2 + 28x}{9 \text{ - } 3x}$$

by the Pythagorean Theorem:

$$3^2 - y^2 = x^2$$

$$9 - [\frac{-3x^2 + 28x}{9 - 3x}] = x^2$$

add fractions

$$\frac{9 * (9 - 3x)}{9 - 3x} - [\frac{-3x^2 + 28x}{9 - 3x}] = x^2$$

$$\frac{81 - 27x}{9 - 3x} + \frac{3x^2 - 28x}{9 - 3x} = x^2$$

$$\frac{3x^2 - 55x + 81}{9 - 3x} = x^2$$

multiply both sides by $[9 - 3x]$

$$3x^2 - 55x + 81 = x^2 * [9 - 3x]$$

$$3x^3 - 6x^2 - 55x + 81 = 0$$

solve for "x*" in this cubic funtion*

I plugged into graphing calculator:

$$x_1 = 4.68354$$

$x_2 = 1.40872$ --- *This is the value we want. It is less than* 3.

$$x_3 = -4.09226$$

Plug into Pythagorean Theorem

$$3^2 - 1.40872^2 = y^2$$

$$y = \sqrt{7.015507}$$

$$= 2.648680$$

I do not claim this is correct. I am only demostrating the power of the Ssine and Scosine.

We start with a unit circle.

It is known that as r (the radius) moves at the different angle the [cos(θ) *r] or "the length of r in the x direction" changes.

The cos decreases between 0 and 90. This information is widely known and documented.

But what would happen if we took a perpendicular (orthogonal) line from the [cos(θ_1) * r] or "the value of "x" at θ_1" and wanted to find how much longer "r" would have to be at "θ_2" to reach the same distance in the "x direction?"

Theorem:

Given θ_1 and θ_2 are within 90 degrees of each other
and $\cos(\theta_1) \geq \cos(\theta_2)$

then:

The length of the segment ("L") (which is at the same angle of θ_2) is given by the equation:

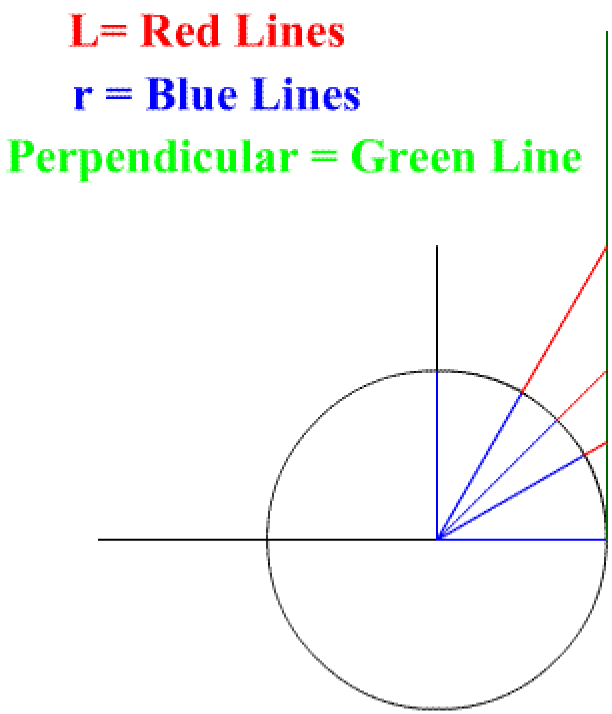
$$[(|\cos(\theta_1)| - |\cos(\theta_2)|) / \cos\theta_2] * r$$

We will call this length or distance "L" or "the length of the segment added to radius "r" to naintain the same horizontal "x distance".

So

"the length of the original segment" or "r" + "the length of the newly found segment L"

$$r + [[(|\cos(\theta_1)| - |\cos(\theta_2)|) / \cos\theta_2] * r]$$


L= Red Lines
r = Blue Lines
Perpendicular = Green Line

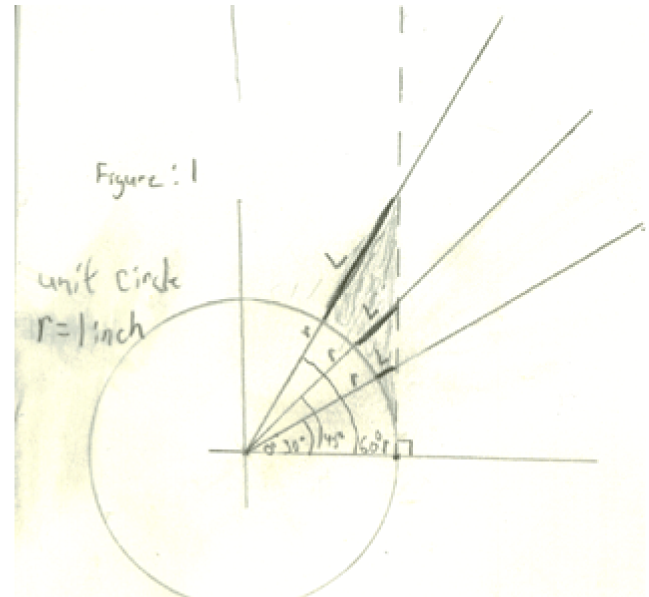


Figure : 1
unit circle
r = 1 inch
L
L
L
r
r
r
30°
45°
60°

This equation can be applied to the other quadrants. Just be sure that the $\cos(\theta_1) \geq \cos(\theta_2)$

We will start by comparing this new length "L" by the special angles of 30, 45, and 60 degress.
On a unit circle with:
let $\theta_1=0$
let θ_2=the current value in degrees
let $L = [(|\cos(\theta_1)| - |\cos(\theta_2)|) / \cos\theta_2] * r$

Value in deg	cos	$\|\cos(\theta_1)\| - \|\cos(\theta_2)\|$	$(\|\cos(\theta_1)\| - \|\cos(\theta_2)\|) / \cos\theta_2$	L	L+R	$\cos(\theta_2)*r + \cos(\theta_2)*L$
0	1	0	0	0	1	0
30	.866	.134	.155	.155	1.155	1.134
45	.707	.293	.414	.414	1.414	1.293
60	.500	.500	1.000	1.000	2.000	1.500
90	0	no value	no value	no value	no value	no value

As θ_2 approaches 90 degrees the length of L and L+R approaches ∞ "infinity"

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Similarly for the Sine:

We start with a unit circle.

It is known that as r (the radius) moves at the different angle the [sin(θ) *r] or "the length of r in the y direction" changes.

But what would happen if we took a perpendicular (orthogonal) line from the [sin(θ_1) * r] or "the value of "y" at θ_1" and wanted to find how much longer "r" would have to be at "θ_2" to reach the same distance in the "x direction?"

Theorem:

Given θ_1 and θ_2 are within 90 degrees of each other
and $\sin(\theta_1) \geq \sin(\theta_2)$

then:

The length of the segment ("L") (which is at the same angle of θ_2) is given by the equation:

$$[(|\sin(\theta_1)| - |\sin(\theta_2)|) / \sin\theta_2] * r$$

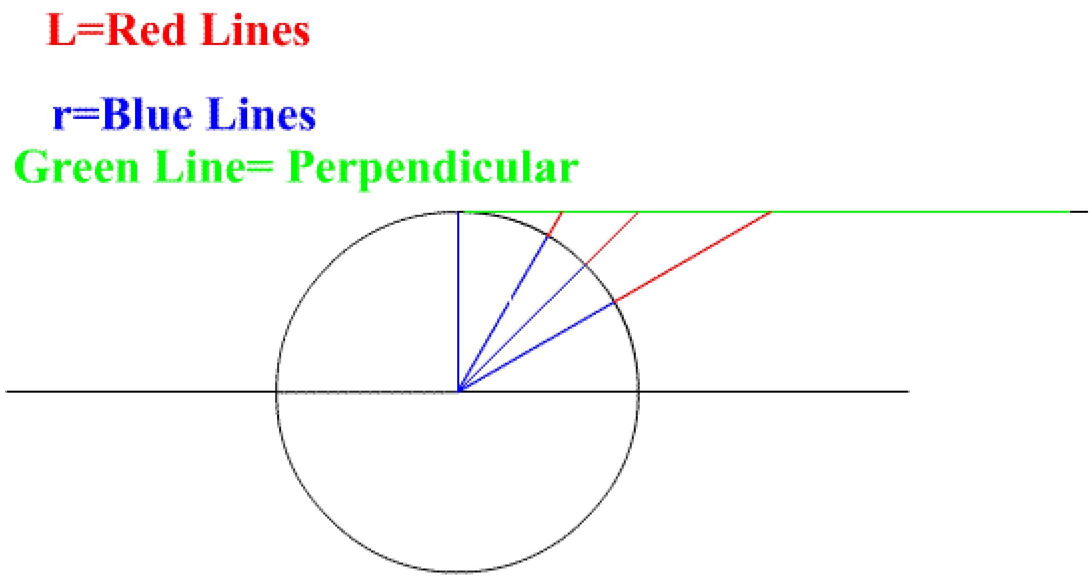
We will call this length or distance "L" or "the length of the segment added to radius "r" to maintain the same vertical "y distance".

So

"the length of the original segment" or "r" + "the length of the newly found segment L"

$$r + [[(|\sin(\theta_1)| - |\sin(\theta_2)|) / \sin\theta_2] * r]$$

This equation can be applied to the other quadrants. Just be sure that the $\sin(\theta_1) \geq \sin(\theta_2)$


L=Red Lines
r=Blue Lines
Green Line= Perpendicular

We will start by comparing this new length “L” by the special angles of 30, 45, and 60 degress.
On a unit circle with:
let θ_1=90 degrees
let θ_2=the current value in degrees
let $L = [(|\sin(\theta_1)| - |\sin(\theta_2)|) / \sin\theta_2] * r$

Value in deg	sin	$\|\|\sin(\theta_1)\| - \|\sin(\theta_2)\|$	$(\|\sin(\theta_1)\| - \|\sin(\theta_2)\|) / \sin\theta_2$	L	L+R	$\sin(\theta_2)*r + \sin(\theta_2)*L$
0	0	no value	no value	no value	no value	no value
30	.500	.500	1.000	1.000	2.000	1.500
45	.707	.293	.414	.414	1.414	1.293
60	.866	.134	.155	.155	1.155	1.134
90	0	1.000	0	0	0	0

As θ_2 approaches 0 degrees the length of L and L+R approaches ∞ “infinity”

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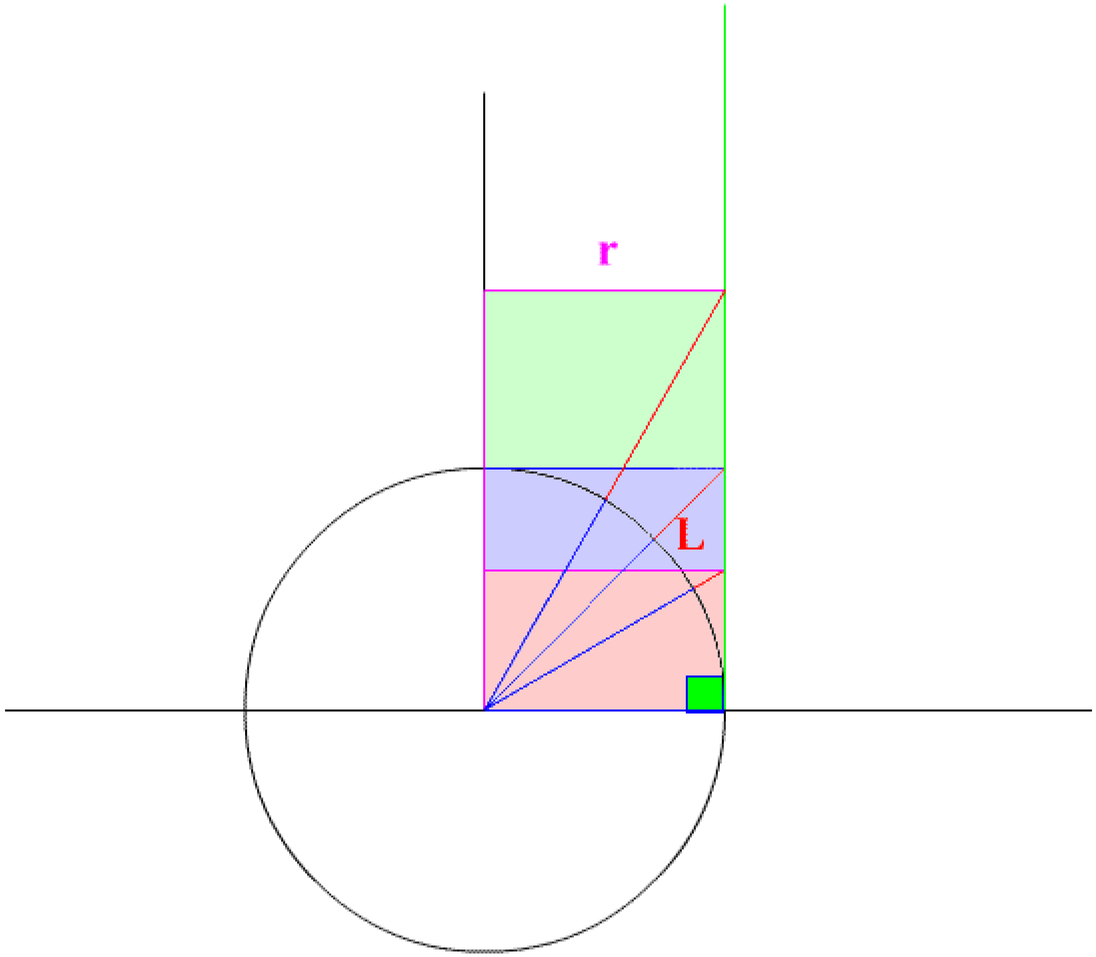
These trigonometric theorems are more complicated the usual cosine and sine functions, but there is a simple rectangle that is created. If the perpendicular is taken from the cosine, the rectangle has a distance in the "x direction" of the cos of θ_1 or a distance of r if the perpendicular was taken from 0 degrees or 180 degrees. The same can be said about the sine. If the perpendicular is taken from the sine, the rectangle has a distance in the "y direction" of the sin of θ_1 or a distance of r if the perpendicular was taken from 90 degrees or 270 degrees.



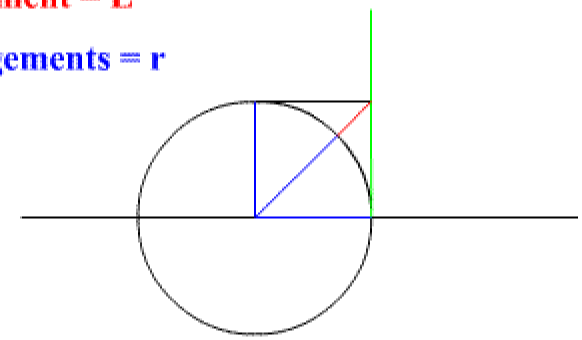
There is a special rectangle formed at 45 degrees with all four sides equal to the radius.

This is a square formed around the diagonal L + r

(where r is the radius and L is the length previously described in the previous equations)

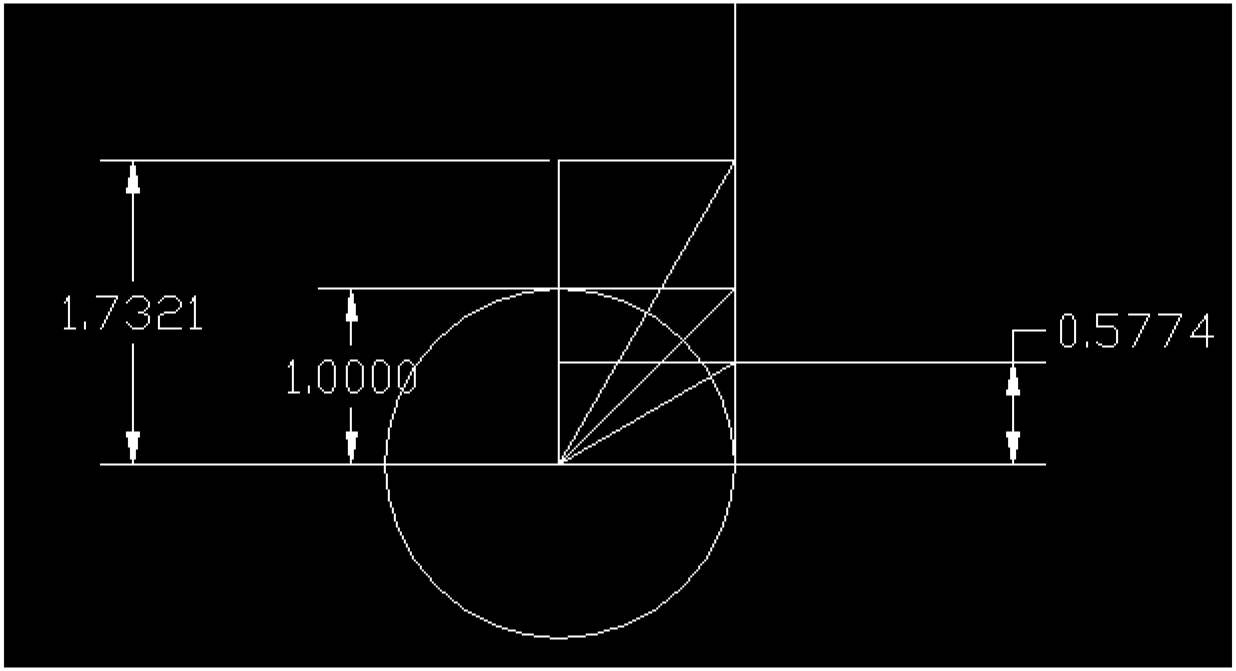
Red segment = L

Blue segements = r

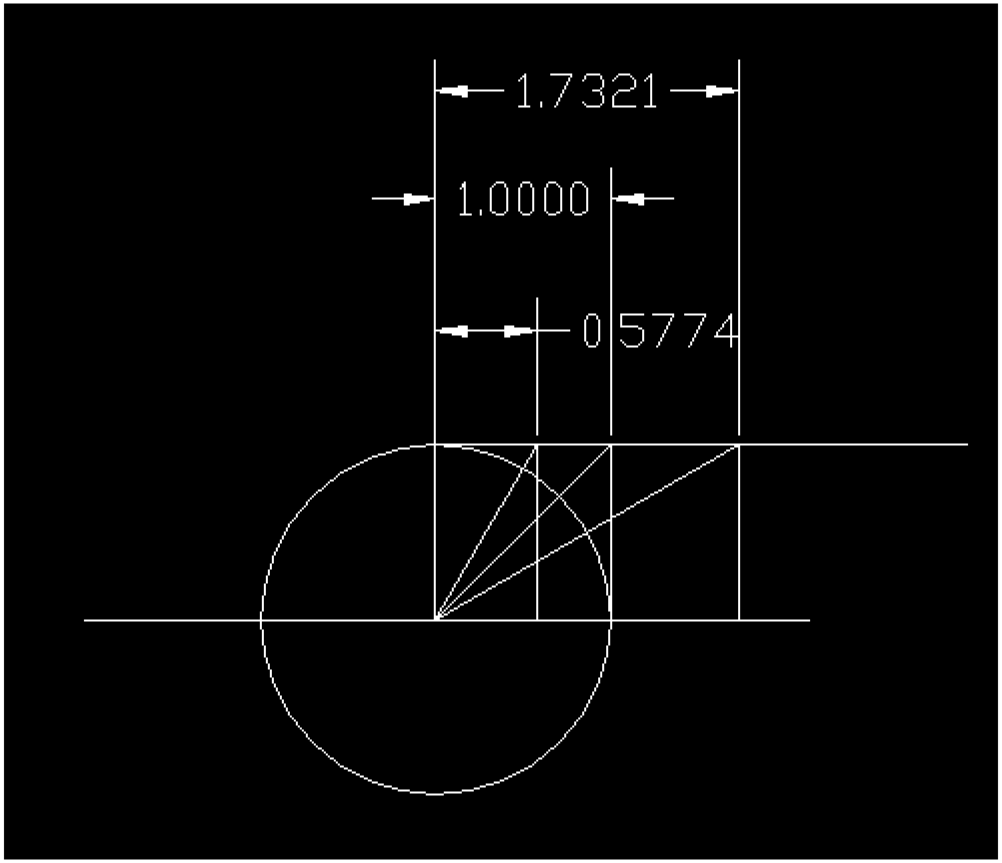


At 45 degrees a special rectangle is formed. Where all 4 sides equal the radius. This happens when the perpendicular is taken from both the sine and cosine!

The ratios (or proportions) of the height "y direction" for L + r for the cosine. As follows:



The ratios (or proportions) of the height "x direction" for L + r for the sine. As follows:



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