
20091116

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For this problem refer to these links:

Trigonometric Parabola Part0001

Part0002

Worked Example

Scosine

would you agree that

$$x + y = 5$$

and

$$x + \frac{y}{3} = 7$$

can be solved by substituting x and y

$$y = 3(7 - x)$$

plug into the first equation

$$x + 3(7 - x) = 5$$

$$x + 21 - 3x = 5$$

$$-2x = -16$$

$$x = 8$$

and plug into

$$y = 3(7 - x)$$

$$y = -3$$

This type of simple substitution is all I am trying to do in the "Trigonometric Parabola" problem.

For my two equations I use the trigonometric pythagorean identity:

$$(\cos x)^2 + (\sin x)^2 = 1$$

and I use an identity that I mathematically derived myself from what I call the Scosine.

(Click Here to see background of Scosine.)

I define "L" as the Scosine as

$$L = \text{Scosine} = \frac{|\cos(\text{first angle}) - \cos(\text{second angle})|}{\cos(\text{second angle})} \cdot \text{radius}$$

but in our problem we know that

$$\cos(\text{first angle}) = 1$$

In the trigonometric parabola the known value (3) in our example is the hypotenuse of the right triangle we are forming. We will define "x" as the length of the horizontal leg that makes up this unknown right triangle.

So with x and an angle with $\cos(\text{first angle}) = 1$

we can write:

$$3 = L + x$$

but since we need to substitute x and y we need an equation that defines y:

$$\text{hypotenuse} = 3 = \frac{1}{L} + y$$

this equation is possible since

$$\frac{1}{\text{Scosine}} = \text{Ssine} = \text{the change in } y \text{ distance of 2 given angles}$$

Example of finding the right triangle that falls on the parabola with a length (hypotenuse in our problem) of 3:

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{x}{3}$$

$$\sin\theta = \frac{y}{3}$$

$$\text{placed into } (\cos x)^2 + (\sin y)^2 = 1$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

The cosine of the right triangle is $\frac{x}{3}$

$$3 = \frac{1}{\frac{1-\frac{x}{3}}{\frac{x}{3}}} + y$$

$$\frac{\frac{x}{3}}{1-\frac{x}{3}} + y = 3$$

$$\frac{\frac{x}{3}}{1-\frac{x}{3}} = 3 - y$$

$$\frac{x}{3} = (1 - \frac{x}{3})(3 - y)$$

$$x = \frac{1}{3} (3 - y - x + \frac{x \cdot y}{3})$$

$$x = 1 - \frac{y}{3} - \frac{x}{3} + \frac{xy}{9}$$

$$3 \cdot (\frac{4}{3} \cdot x - 1) = -y + \frac{xy}{3}$$

$$4x - 1 = -y + \frac{xy}{3}$$

$$\frac{3 \cdot (4x - 1)}{x \cdot y} = -y$$

$$\frac{3 \cdot (4x - 1)}{x} = -y^2$$

$$y^2 = -1 \cdot \left[\frac{3 \cdot (4x - 1)}{x} \right]$$

as previously derived

$$y = \sqrt{(3 - x)(3 + x)}$$

$$y = \sqrt{3^2 - x^2}$$

substitute

$$[\sqrt{3^2 - x^2}]^2 = -1 \cdot \left[\frac{3 \cdot (4x - 1)}{x} \right]$$

$$x^2 - 3^2 = \frac{3 \cdot (4x - 1)}{x}$$

$$x^3 - 9x = 3 \cdot (4x - 1)$$

$$3x^3 - 27x = 4x - 1$$

$$3x^3 - 27x + 1 = 4x$$

$$3x^2 + \frac{1}{x} - 27 = 4$$

$$3x^2 + \frac{1}{x} - 23 = 0$$

Plug into the quadratic equation:

$$\frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2a}$$

$$\frac{-1 \pm \sqrt{-1^2 - 4 \cdot 3 \cdot -23}}{2 \cdot 3}$$

$$\frac{-1 \pm \sqrt{1 + 276}}{6}$$

$$\frac{-1 + \sqrt{277}}{6} \text{ or } \frac{-1 - \sqrt{277}}{6}$$

$$x = 2.6072 \text{ or } x = -2.9405$$

Plug into right triangle trigonometric ratios:

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \text{ on a right triangle}$$

$$\cos\theta = \frac{2.6072}{3}$$

$$\theta = \cos^{-1}\left[\frac{2.6072}{3}\right]$$

$$\theta = 29.6496 \text{ degrees}$$

So we basically have a 30---60 triangle and can use this info to find where an arc of π radians intersects a parabola with the distance from the apex equal to our Prime number

I am currently getting values for a triangle that works. But it needs to be proved that it is the specific triangle I am looking for. I have been told the equation is not quadratic but cubic. However I still feel the problem is a step in the right direction.